Physically-Based Hydraulic Erosion

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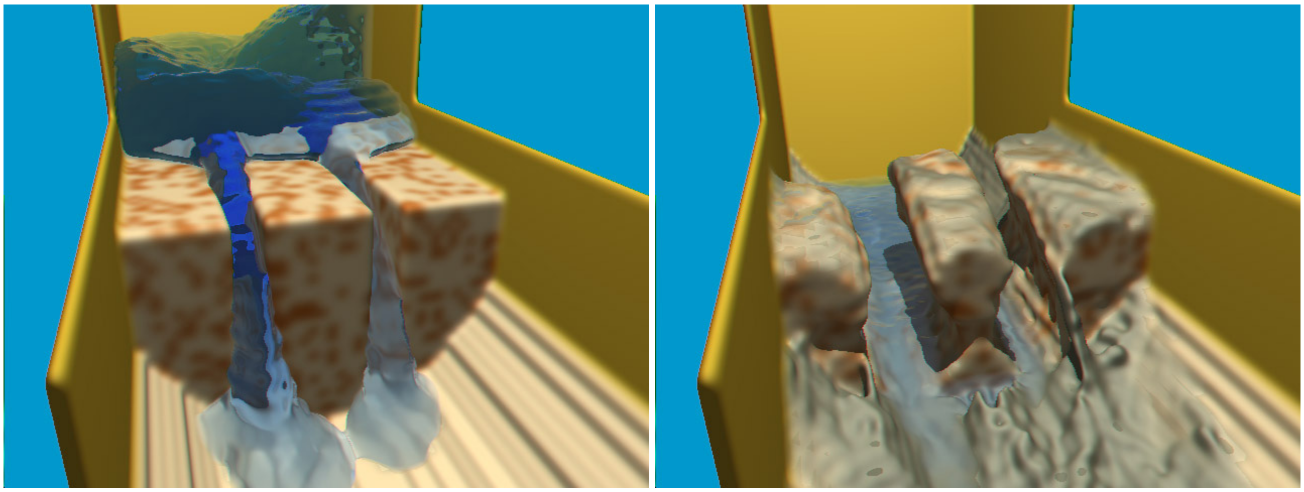

Figure 1: Example of an eroded scene

Abstract

Terrain morphology has been on the radar of Computer Graphics for more than twenty years and various techniques for its modeling have been presented. These approaches range from fractal and multifractal terrain generation to physically-based models of erosion and weathering. Of all the weathering phenomena that can be observed in Nature, hydraulic erosion has the most visual importance. We will focus on hydraulic erosion in our presentation and we will describe a technique that is rooted in a generalized solution to modeling hydraulic erosion using ideas from fluid mechanics. The simulations show the terrain morphogenesis and can be used for animations as well as for static scene models generation.

CR Categories: I.3.3 [Computer Graphics]: Picture/Image Generation – Display algorithms— [I.3.7]: Computer Graphics—Three Dimensional Graphics and Realism – Virtual Reality

Keywords: Weathering, Erosion, Terrain Morphology

1 Introduction

Weathering, erosion, changes in appearance, morphology, all of these phenomena have been in the focus of computer graphics for more than twenty tears. Algorithms and methods that try to capture changes in shape or appearance, are not only hard to generalize, but are difficult to classify because some fail to fall within the two basic categories, modeling and rendering.

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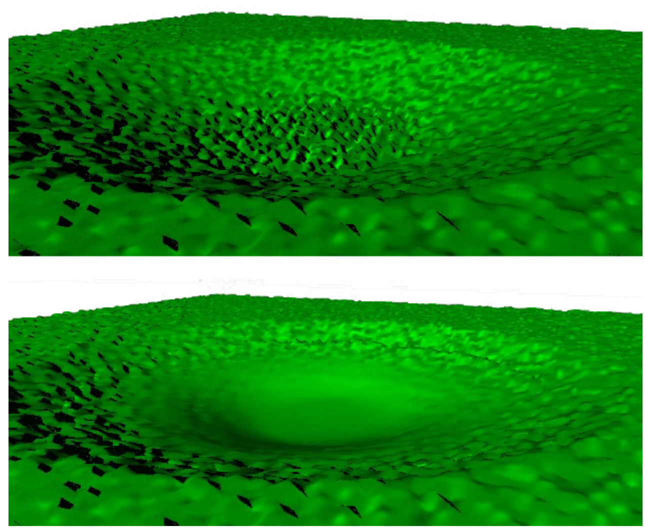

Figure 2: Erosion by sediment deposition. Soil that has dissolved to a pool of water and deposited in a different location

The most interesting problem is the terrain shape modeling. Nowadays, the *de facto* standard techniques exploit fractal methods [Deussen et al. 2003] that provide fast solutions and also different levels of detail. Fractals provide the final terrain shape description. They do not depict the terrain evolution over time. To further describe morphological changes erosion simulations need to be defined and applied.

Visual models of terrains have been represented as triangular meshes, but in this representation it is difficult to apply to terrain morphology. That is why the memory consuming, *regular height fields* are frequently used. For example one of the first papers [Musgrave et al. 1989] uses regular height fields to describe thermal weathering and splashing soil by water. A simple diffusion-based model is used as means for underlying transportation. Regular height fields are normally used for off-line preparation of terrains that are then converted to triangular meshes and rendered. There are many different algorithms for terrain morphology generation working with regular height fields. Some deal with sand and soils [Beneš, B. and Arriaga, X. 2005; Li and Moshell 1993; Koichi and Nishita 2000; Koichi and Nishita 2003; Roudier 1993; Sumner et al. 1999], others focus on hydraulic erosion [Beneš and Forsbach 2002; Chiba et al. 1998; Neidhold, B. and Wacker, M. and Deussen, O. 2005], and some present purely *ad hoc* solution [Ito et al. 2003; Kelley et al. 1988; Nagashima 1997; Pickover 1995; Stachniak, S. and Stuerzlinger, W. 2005].

The majority of the previous work focuses on large-scale phenomena such as rivers, dunes, deserts, etc. This is possible due to the fact that they use only a two dimensional solution to the erosion problem. The application area is limited to large-scale erosion. A logical step to realistic erosion models is to utilize volumetric morphology representation such as [Varadhan and Mueller 2003; Zhu and Bridson 2005]. The complete volumetric erosion presents a challenging problem. Data necessary to describe terrains is enormous, therefore only a small-scale erosion can be modeled in this way. We briefly describe one attempt at a full three dimensional erosion algorithm in this paper.

We have proposed a complete solution of volumetric erosion by means of fluid dynamics in [Beneš et al. 2006].

2 Hydraulic Erosion Using Fluid Dynamics

Erosion is a three-step process. First, the boundary between the two layers is damaged. The materials between the layers blend and are then transported. Eventually the material is deposited in a different location.

2.1 Fluid Dynamics

The key factor in the process of hydraulic erosion is the transportation of water that is fully described by the Navier-Stokes equations. They provide velocity and pressure fields and their dynamics [Enright et al. 2001; Foster and Fedkiw 2001; Stam 1999]

$$\nabla \cdot \boldsymbol{u} = 0 \quad (1)$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f} \quad (2)$$

In the equations ρ is the liquid density, ν is its viscosity, and $\boldsymbol{f}$ is the vector of external forces. Equation (1) reflects the incompressibility and mass conservation of liquid and (2) expresses the conservation of momentum.

A practical approach used in Computer Graphics to the solution of the Navier-Stokes equations [Foster and Fedkiw 2001] is based on a regular discrete representation of the three-dimensional space. Each voxel is classified to fall into one of the three categories:

1. `FULL` – Voxel is full of water.
2. `EMPTY` – Voxel does not contain anything except air.
3. `MAT` – Voxel contains some material.

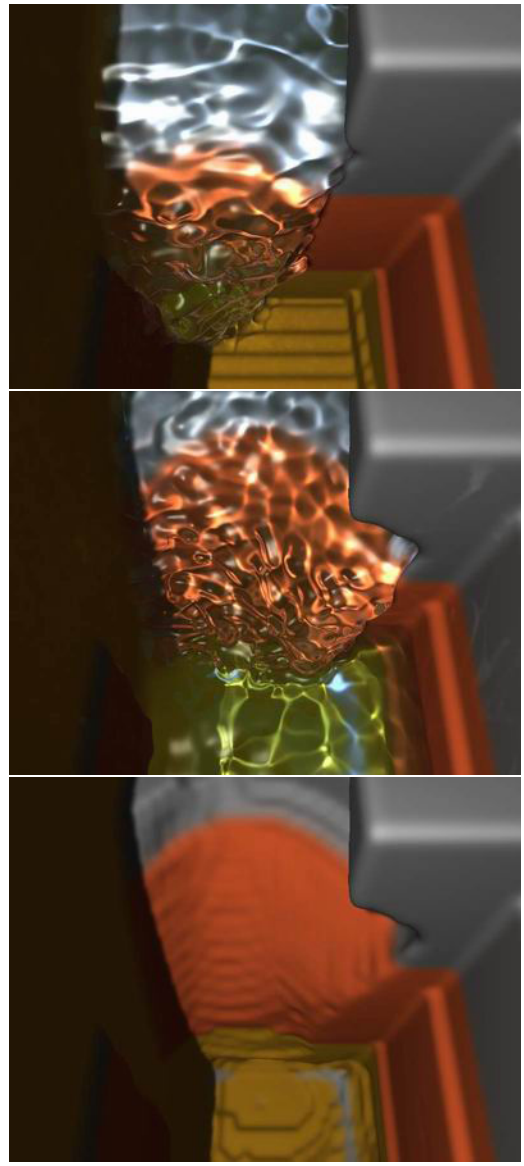

Figure 3: Sequence of images from the animation of a receding waterfall

2.2 Voxel State Changes

We have coupled this mode with the sediment transportation equation commonly used in hydrology [Langendoen 2003]. To do this the concept of the `FULL`, `EMPTY`, and `MAT` voxels are extended by the state changes.

First we define a scalar variable m that defines the amount of the material in a `FULL` voxel. This represents the quantity of the captured material. The `FULL` voxel may contain any material. If it does contain material, the material may be dissolved in liquid, up to a saturation point.

The `MAT` voxels with $0 < m < 1$ are located on the boundary between the water and the material, or the air and the material. A water voxel

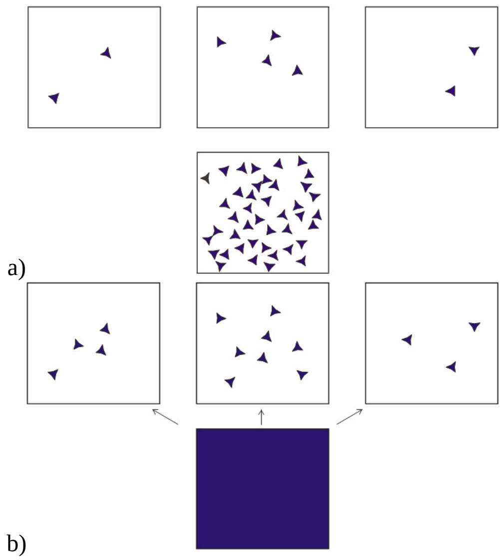

Figure 4: The process of deposition. The saturated cell (in the lower part of Figure a) changes its state from FULL⇒MAT and the excess of the material is distributed among the neighbors

cannot carry more sediment than its saturation point. Similarly, a MAT voxel that is not on a boundary has $m = 1$.

A voxel can change its state by either of the following two transitions [Beneš et al. 2006]:

- FULL $\Rightarrow$ MAT for the material deposition and
- MAT $\Rightarrow$ FULL to erode.

Deposition always occurs towards the bottom of the voxel. A FULL voxel is full of water as long as the amount of dissolved material is smaller than its saturation capacity. When the voxel is oversaturated it changes its state through the transition FULL $\Rightarrow$ MAT. The excess material is redistributed to all the adjacent voxels that are FULL. This process is described in Figure 4.

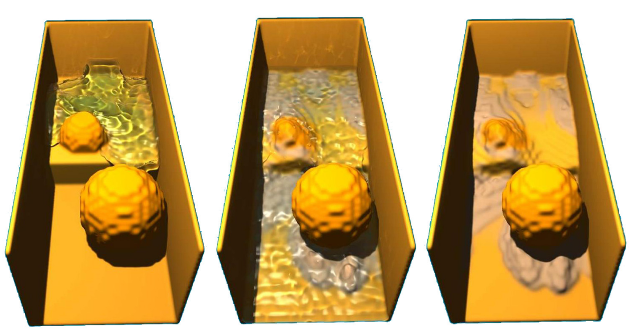

Figure 5: An example of hydraulic erosion

2.3 Erosion and Deposition

The erosion/deposition model is inspired by the [Langendoen 2003] model used in hydrology. Suppose C is the concentration of the sediment mass, E is the erosion rate, and D represents the deposition rate. The sediment transportation equation is given by

$$\frac{\partial C}{\partial t} + (\boldsymbol{u} \cdot \nabla) C = E - D \tag{3}$$

E and D have different values for a cohesive or cohesionless material. The more common case of a cohesive material is described as

$$\begin{aligned} E &= e\left(\frac{\tau}{\tau_{ce}} - 1\right) \\ D &= d\omega\left(1 - \frac{\tau}{\tau_{cd}}\right) \end{aligned} \tag{4}$$

Here $e = 0.01$ is the erosion-rate constant, τ is the bed shear stress and is a function of the geometry of the object. τ_{ce} is the shear stress strength of the material, $d = 0.01$ is the deposition-rate constant, τ_{cd} is the shear stress strength of the material below which the particles start to deposit, and $\omega = 0.2$ is the fall velocity within the liquid.

The term $1 - \tau/\tau_{cd}$ describes the probability that the particle will stick with the material and not re-enter into the flow. In our experiments, we have used the values $\tau_{ce} = \tau_{cd} = 10$. If $f = 1/2$ denotes the material friction and $\rho = 0.001$, the value of τ is given in terms of viscosity v and density ρ by

$$\tau = \frac{f \rho v^2}{2} \tag{5}$$

Equation 3 also reflects the material transport that is described implicitly by the Navier-Stokes equations in our model. Then

$$\frac{dC}{dt} = E - D. \tag{6}$$

Solving Equation 6 by Euler's method for each time interval Δt, we have

$$\begin{aligned} C^{n+1} &= C^* + \Delta t (E^* - D^*) \\ E^* &= e\left(\frac{\frac{1}{2}\left(\tau^{n+1} + \tau^n\right)}{\tau_{ce}} - 1\right) \\ D^* &= d\omega\left(1 - \frac{\frac{1}{2}\left(\tau^{n+1} + \tau^n\right)}{\tau_{cd}}\right) \end{aligned} \tag{7}$$

2.4 Scene Compression

The erosion simulation depicted in the full three-dimensional representation is far from being real-time, and which is why the scenes should be recorded for further visualization. We store the pressure and the velocity fields, the voxel status, and the amount of dissolved material. The problem is that a typical animation of 700 frames of a scene of 300×300 voxels requires about 300 GB of the storage space.

The compression must be lossless and must allow for scrolling forward and backward in the animations. We have proposed such a compression scheme in [Beneš, B. and Těšínský, V. 2005].

The scene contains layers of air, water, and soils and is great candidate for compression. Also, the temporal coherence in the animated sequences is significant. We have developed a key-framing differential scheme to store the animated sequences.

The reference key-frames are stored every twenty frames to allow for fast scrolling and previewing of the animations. The key-frames are stored using run length encoding (RLE) compression scheme and the compression ratio is 1:30. The difference scenes are stored relatively to the key-frames and their compression ratio is about 1:400. The overall compression factor of the animated sequences is approximately 1:100. The compression factor worsens with complicated scenes that include complex free-levels of water and uneven boundaries between the water and the materials.

2.5 Rendering

Our system supports two different rendering methods. The above described compression scheme also stores the free-level of water that is detected as a 3D mesh by the marching-cubes algorithm. This way the scene can be previewed in real time. Snapshot of the application that uses GLUI for the user interface and OpenGL for displaying the scene is in Figure 6.

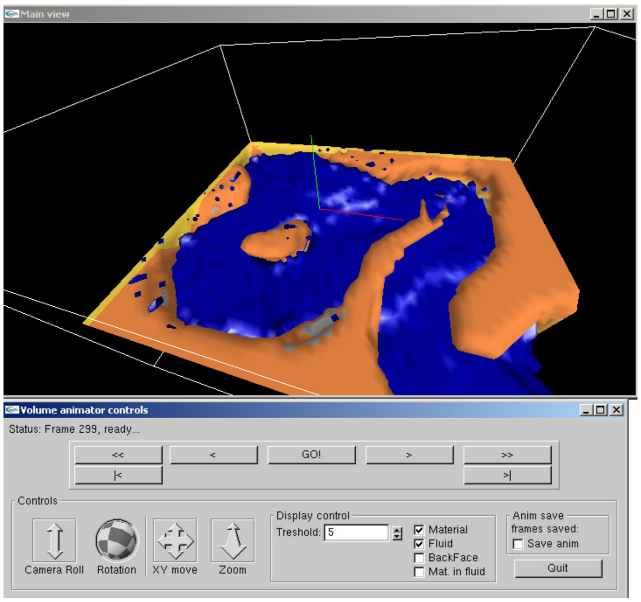

Figure 6: The previewer uses OpenGL to display the scene. The free level of water is calculated by the marching cubes algorithm

A complete photon mapper was developed to display the scenes correctly. This standalone application allows for camera manipulation and animation and displays caustics and refractions.

The interface of this application displays raytraced previews at interactive framerates. It first benchmarks the computer and based on the test it decides the way the rendering must be simplified. It is really useful for scene manipulations and camera positioning. An example of a scene rendered by the system can be seen in Figure 5.

The solution provides not only the static three-dimensional models of eroded scenes, but also high quality animations of the erosion process such as meander break, receding waterfalls (Figure 3), etc.

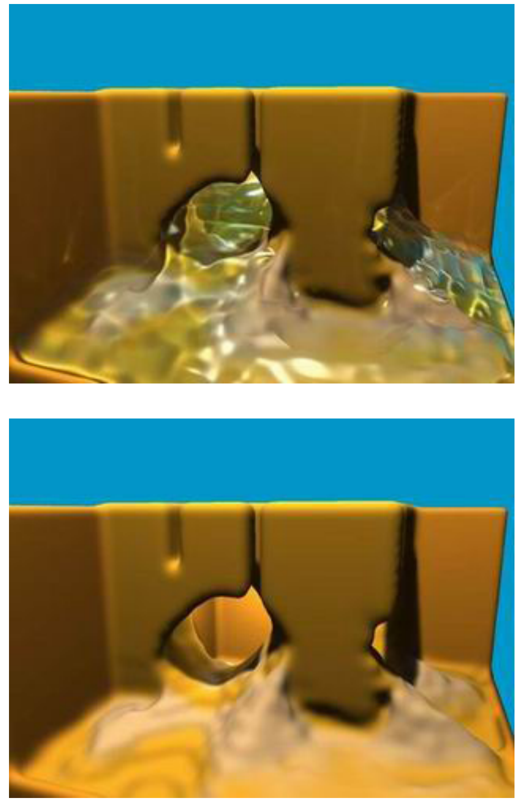

Figure 7: Virtual maze before and after hydraulic erosion

3 Future Work

We have described the first step to the physically-based hydraulic erosion. The results are visually plausible and the method can be applied in computer animations or in scene modeling. There are still many problems to be solved.

One of the problems is the speed of the simulation. The actual calculation of three minutes of a voxel space in resolution 300^3 takes several hours on a computer with a 3*GHz* CPU. One of the possible methods of increasing the speed could be using GPU. Another possible way is not to use voxelization of the space and to apply techniques from particle based simulation of fluid dynamics such as [Wei et al. 2003].

Another interesting problem is the scene size. We want to save the scenes with all the information for further displaying. But then we deal with a huge amount of data that need some kind of compression. A partial solution to this problem can be found in [Beneš, B. and Těšínský, V. 2005], but a better solution remains as a future consideration.

The dissolved material visualization is another interesting problem.

Yet, another important problem is the description of a user-driven or interactive erosion simulation. A majority of the discussed approaches are just ”run and see” techniques, where the user's interaction is actually limited to setting the input parameters of the simulation. A better interaction is necessary. Users should be able to interact with the scene, change the model, change the flow of the liquid, etc. This is impossible with the computational power of computers on one hand and the complexity of the methods on the other hand.

The relation between the low and large scale simulations poses another problem to be solved. Different algorithms working on different scales are available. Very low-level details, such as [Dorsey et al. 1999] are available, or very large-scale techniques are used in Geographic Information Systems. Determining the relation between these different scales is another problem that needs to be addressed. An algorithm coupling small and large scales needs to be found.

Additional images and animations can be found at www2.tech.purdue.edu/Cgt/facstaff/bbenes/private/benes.htm.

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References

BENEŠ, B., AND FORSBACH, R. 2002. Visual simulation of hydraulic erosion. *Journal of WSCG 10*, 1, 79–86.

BENEŠ, B. AND TĚŠÍNSKÝ, V. 2005. A Compression Scheme for Volumetric Animations of Running Water. In *A Compression Scheme for Volumetric Animations of Running Water*, vol. 32, 27–39.

BENEŠ, B. AND ARRIAGA, X. . 2005. Table Mountains by Virtual Erosion. In *Proceedings of Eurographics Workshop on Natural Phenomena*, vol. 1, 33–40.

BENEŠ, B., TĚŠÍNSKÝ, V., HORNYŠ, J., AND BHATIA, S. 2006. Hydraulic erosion. *Computer Animation and Virtual Worlds 17(2)*, 99–108.

CHIBA, N., MURAOKA, K., AND FUJITA, K. 1998. An erosion model based on velocity fields for the visual simulation of mountain scenery. *The Journal of Visualization and Computer Animation 9*, 185–194.

DEUSSEN, O., FEDKIW, R., MUSGRAVE, F., PRUSINKIEWICZ, P., AND STAM, J. 2003. Ebert, D.D. organizer, Course 41: Simulating Nature: Realistic and Interactive Techniques. *Siggraph 2003 Course Notes*, 1–301.

DORSEY, J., EDELMAN, A., JENSEN, H. W., AND PEDERSEN, H. K. 1999. Modeling and Rendering of Weathered Stone. In *Proceedings of SIGGRAPH '99*, ACM Press / ACM SIGGRAPH, vol. 25(4) of *Computer Graphics Proceedings, Annual Conference Series*, ACM, 225–234.

ENRIGHT, D., MARCHNER, S., AND FEDKIW, R. 2001. Animation and rendering complex water surfaces. In *Proceedings of SIGGRAPH 2002*, ACM Press / ACM SIGGRAPH, J. F. Hughes, Ed., Computer Graphics Proceedings, Annual Conference Series, ACM, 736–744.

FOSTER, N., AND FEDKIW, R. 2001. Practical animation of liquids. In *Proceedings of SIGGRAPH 2001*, ACM Press / ACM SIGGRAPH, E. Fiume, Ed., Computer Graphics Proceedings, Annual Conference Series, ACM, 23–30.

ITO, T., FUJIMOTO, T., MURAOKA, K., AND CHIBA, N. 2003. Modeling rocky scenery taking into account joints. In *Computer Graphics International*, 244–247.

KELLEY, A. D., MALIN, M. C., AND NIELSON, G. M. 1988. Terrain simulation using a model of stream erosion. 263–268.

KOICHI, O., AND NISHITA, T. 2000. A Method for Modeling and Rendering Dunes with Wind-ripples. In *Proceedings of Pacific Graphics'00*, 427–428.

KOICHI, O., AND NISHITA, T. 2003. Virtual sandbox. In *Proceedings of Pacific Graphics'03*, IEEE Computer Society, 252–260.

LANGENDOEN, E. 2003. Concepts - conservational channel evolution and pollutant transport system: Stream corridor version 1.0. Tech. rep., US Department of Agriculture, Agricultural Research Service.

LI, X., AND MOSHELL, M. 1993. Modeling Soil: Realtime Dynamic Models for Soil Slippage and Manipulation. In *Proceedings of SIGGRAPH'93*, vol. 27(4) of *Annual Conference Series*, 361–368.

MUSGRAVE, F. K., KOLB, C. E., AND MACE, R. S. 1989. The synthesis and rendering of eroded fractal terrains. In *SIGGRAPH '89: Proceedings of the 16th annual conference on Computer graphics and interactive techniques*, ACM Press, New York, NY, USA, 41–50.

NAGASHIMA, K. 1997. Computer Generation of Eroded Valley and Mountain Terrains. *The Visual Computer 13*, 456–464.

NEIDHOLD, B. AND WACKER, M. AND DEUSSEN, O. 2005. Interactive physically based Fluid and Erosion Simulation. In *Proceedings of Eurographics Workshop on Natural Phenomena*, vol. 1, 25–32.

PICKOVER, C. 1995. Generating Extraterrestial Terrain. *IEEE Computer Graphics and Applications 17*, 18–21.

ROUDIER, P. 1993. Landscapes synthesis achieved through erosion and deposition process simulation. *Computer Graphics Forum 12*, 3, 375–383.

STACHNIAK, S. AND STUERZLINGER, W. 2005. An Algorithm for Automated Fractal Terrain Deformation. In *Proceedings of Computer Graphics and Artificial Intelligence*, vol. 1, 64–76.

STAM, J. 1999. Stable fluids. In *Proceedings of SIGGRAPH 99*, ACM SIGGRAPH/ Addison Wesley Longman, Computer Graphics Proceedings, Annual Conference Series, 121–128.

SUMNER, R. W., O'BRIEN, J. F., AND HODGINS, J. K. 1999. Animating Sand, Mud, and Snow. *Computer Graphics Forum 18*, 1, 17–26.

VARADHAN, H., AND MUELLER, K. 2003. Volumetric ablation rendering. In *VG '03: Proceedings of the 2003 Eurographics/IEEE TVCG Workshop on Volume graphics*, ACM Press, 53–60.

WEI, X., ZHAO, Y., FAN, Z., LI, W., YOAKUM-STOVER, S., AND KAUFMAN, A. 2003. Blowing in the wind. In *Proceedings of the 2003 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, Eurographics Association, ACM, 75–85.

ZHU, Y., AND BRIDSON, R. 2005. Animating sand as a fluid. *ACM Trans. Graph. 24*, 3, 965–972.